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High-frequency surface waves on a semi-bounded warm current plasma

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Abstract. This paper investigates the dispersion relation and the time and space damping rates of the high-frequency surface waves propagating on a warm homogeneous non-isothermal current plasma bounded by a vacuum. The description is done on the basis of a kinetic plasma theory under the specular reflection condition for particles at the plasma–vacuum boundary.

1. Introduction

In a number of works (Romanov 1964, Gorbatenko and Kurilko 1964, Kondratenko 1965, Romanov 1968, Guernsey 1969, Kondratenko 1972, Barr and Boyd 1972) on the basis of a kinetic theory the dispersion and the time damping rate of the high-frequency surface waves propagating on the plasma–vacuum boundary have been obtained. The electrostatic and electrodynamic considerations that have been treated include a plasma with different geometries (semi-bounded plasma, layer or cylinder). The boundary conditions used are perfectly reflecting walls or a diffuse plasma–vacuum boundary.

Recently, in some experimental work on surface waves, results on the propagation of surface waves in a current plasma have been reported (Aničin *et al* 1973). They have established that the downstream wavenumbers are smaller than the upstream wavenumbers and the downstream wave is less attenuated than the upstream wave. The present theoretical paper explains these results. Here we obtain the dispersion relation and the time and space damping rates for high-frequency surface waves. We use a warm non-isothermal current plasma model which is very similar to laboratory discharge plasmas. To simplify the theoretical treatment, we examine a homogeneous plasma filling up a half-space and bounded by a vacuum.

2. Formulation of the problem

Let the half-space $x > 0$ be occupied by a warm homogeneous two-component current plasma and the plane $x = 0$ be the boundary plasma–vacuum surface. We assume that the electrons and the ions of the plasma drift in opposite directions and the velocity of their directed motion is parallel to the boundary surface $x = 0$. As we will investigate the propagation of high-frequency waves only, we neglect directed motion of the ions and consider that the electrons drift with the same velocity V. This allows us to examine the electron component of the plasma as a beam penetrating through the immovable background of ions. We suppose that a narrow transition layer Δx exists at the plasma–vacuum surface just to $x = 0$, in which the plasma number density changes to 0. Really this layer is a Debye plasma length in extent. Since we shall be interested only in waves with wavelengths larger than the thickness of this layer (in this case the Landau damping of the propagating longitudinal waves is not strong) we can assume the plasma–vacuum surface is sharp enough.

In order to obtain the dispersion equation for the high-frequency surface waves in an electrostatic approximation, we describe the dynamics of the plasma by the linearized Vlasov and Poisson equations:

$$\frac{\partial f_{1\alpha}}{\partial t} + \boldsymbol{v} \cdot \nabla_r f_{1\alpha} + \frac{e_\alpha}{m_\alpha} \boldsymbol{E} \cdot \nabla_v f_{0\alpha} = 0$$
$$\operatorname{div} \boldsymbol{E} = 4\pi\rho(x, \boldsymbol{R}_{\|}) \tag{1}$$

where e_α and m_α are the charges and the masses of the particles of type α respectively (α = e, i), $\boldsymbol{E}$ is the electric field of the perturbation in the plasma, $f_{1\alpha} = f_{1\alpha}(x, \boldsymbol{R}_{\|}, v_x, \boldsymbol{v}_{\|}, t)$ are the perturbations in the velocity distribution functions of the particles, where x, $\boldsymbol{R}_{\|}$ and v_x, $\boldsymbol{v}_{\|}$ are the position variables and the velocity components of the particles parallel to the axis Ox and the boundary surface $x = 0$ respectively; $f_{0\alpha} = f_{0\alpha}(v_x, \boldsymbol{v}_{\|})$ are the equilibrium distribution functions and

$$\rho(x, \boldsymbol{R}_{\|}) = \sum_\alpha \rho_\alpha(x, \boldsymbol{R}_{\|}) = \sum_\alpha e_\alpha \int \mathrm{d}\boldsymbol{v} f_{1\alpha}(x, \boldsymbol{R}_{\|}, v_x, \boldsymbol{v}_{\|}, t) \tag{2}$$

is the perturbation of the plasma charge density.

We assume that the equilibrium velocity distribution of the particles is maxwellian taking into account the directed electron velocity:

$$f_{0\mathrm{e}}(\boldsymbol{v}) = n_\mathrm{e}\left(\frac{m_\mathrm{e}}{2\pi T_\mathrm{e}}\right)^{3/2} \exp\left(-\frac{m_\mathrm{e}}{2T_\mathrm{e}}(\boldsymbol{v} - \boldsymbol{V})^2\right)$$
$$f_{0\mathrm{i}}(\boldsymbol{v}) = n_\mathrm{i}\left(\frac{m_\mathrm{i}}{2\pi T_\mathrm{i}}\right)^{3/2} \exp\left(-\frac{m_\mathrm{i}}{2T_\mathrm{i}}\boldsymbol{v}^2\right) \tag{3}$$

where T_α is the temperature of the particles of type α ($T_\mathrm{i} \ll T_\mathrm{e}$) and $n_\mathrm{e} = n_\mathrm{i} = n_0$ are the equilibrium number densities of the electrons and the ions.

In accordance with the geometry of the system the perturbed electrostatic field $\boldsymbol{E}$ may be written in the form:

$$\boldsymbol{E} = -\operatorname{grad} \phi \tag{4}$$

where

$$\phi(x, \boldsymbol{R}_{\|}) = \int_0^\infty \int_{-\infty}^\infty \frac{\rho(x', \boldsymbol{R}'_{\|})}{[(x-x')^2 + (\boldsymbol{R}_{\|} - \boldsymbol{R}'_{\|})^2]^{1/2}} \,\mathrm{d}x' \,\mathrm{d}^2\boldsymbol{R}'_{\|}$$

is the potential of the plasma perturbation.

The boundary condition for the problem is perfect reflection of the particles from the plasma–vacuum boundary (the plane $x = 0$):

$$f_{1\alpha}(0, \boldsymbol{R}_{\|}, -v_x, \boldsymbol{v}_{\|}, t) = f_{1\alpha}(0, \boldsymbol{R}_{\|}, v_x, \boldsymbol{v}_{\|}, t). \tag{5}$$

The boundary condition (5) can be applied in spite of the presence of a current in the plasma as we assume that the drift electron velocity $\boldsymbol{V}$ is parallel to the plane $x = 0$

and, therefore, the condition for perfectly reflecting walls is satisfied from the equilibrium distribution functions of the particles taken in the form (3).

3. Solution of the problem

Following the method proposed by Guernsey (1969), we resolve the system (1) at the assumptions (2), (3), (4) applying a Fourier transformation with respect to the position variables $\boldsymbol{R}_{\|}$ and $\boldsymbol{R}'_{\|}$ and a Laplace transformation with respect to the time t. In order to be able to apply a Fourier transformation with respect to x, we extend the definition range of the function

$$\tilde{f}_{1\alpha}(x, \boldsymbol{k}_{\|}, v_x, \boldsymbol{v}_{\|}, \omega) = \int_0^{\infty} \mathrm{d}t \exp(\mathrm{i}\omega t) \int \mathrm{d}^2\boldsymbol{R}_{\|} \exp(-\mathrm{i}\boldsymbol{k}_{\|} \cdot \boldsymbol{R}_{\|}) f_{1\alpha}(x, \boldsymbol{R}_{\|}, v_x, \boldsymbol{v}_{\|}, t) \tag{6}$$

which is the Fourier–Laplace transform of the function $f_{1\alpha}(x, \boldsymbol{R}_{\|}, v_x, \boldsymbol{v}_{\|}, t)$ in the region $x < 0$, in such a way that the new function $\tilde{f}_{1\alpha}$ satisfies the equation for $\tilde{f}_{1\alpha}$ with $x > 0$ and the boundary condition at $x = 0$. This can be easily made if one assumes that the plane $x = 0$ is a mirror surface and $\tilde{f}_{1\alpha}$ are determined by the equalities:

$$\tilde{f}_{1\alpha}(-x, \boldsymbol{k}_{\|}, v_x, \boldsymbol{v}_{\|}, \omega) = \tilde{f}_{1\alpha}(x, \boldsymbol{k}_{\|}, -v_x, \boldsymbol{v}_{\|}, \omega). \tag{7}$$

For the Fourier transform of $\tilde{f}_{1\alpha}$:

$$F_{1\alpha}(k_x, \boldsymbol{k}_{\|}, v_x, \boldsymbol{v}_{\|}, \omega) = \int_{-\infty}^{\infty} \mathrm{d}x \exp(-\mathrm{i}k_x x) \tilde{f}_{1\alpha}(x, \boldsymbol{k}_{\|}, v_x, \boldsymbol{v}_{\|}, \omega) \tag{8}$$

we obtain the following expression:

$$F_{1\alpha}(k_x, \boldsymbol{k}_{\|}, v_x, \boldsymbol{v}_{\|}, \omega) = \frac{1}{\mathrm{i}(\boldsymbol{k} \cdot \boldsymbol{v} - \omega)} \left[G_\alpha(\boldsymbol{k}, \boldsymbol{v}) + \frac{2\pi \mathrm{i} e_\alpha}{m_\alpha k^2} \left(2\rho(\boldsymbol{k}) - \frac{k_{\|}}{\pi} \int_{-\infty}^{\infty} \mathrm{d}k'_x \frac{\rho(\boldsymbol{k}')}{k'^2} \right) \left(\boldsymbol{k} \cdot \frac{\partial f_{0\alpha}}{\partial \boldsymbol{v}} \right) \right] \tag{9}$$

where

$$\rho(\boldsymbol{k}) = \sum_\alpha e_\alpha \int_{-\infty}^{\infty} \mathrm{d}v_x \int \mathrm{d}^2\boldsymbol{v}_{\|} \, F_{1\alpha}(k_x, \boldsymbol{k}_{\|}, v_x, \boldsymbol{v}_{\|}, \omega),$$

k_x and $\boldsymbol{k}_{\|}$ are the wavevector components parallel to the axis Ox and the boundary surface $x = 0$ respectively and $G_\alpha(\boldsymbol{k}, \boldsymbol{v})$ is the Fourier transform of the perturbation at the moment $t = 0$.

One can see from the expression obtained for the Fourier–Laplace transform of the electric charge plasma density:

$$\rho(\boldsymbol{k}, \omega) = -\frac{\mathrm{i}}{\Delta(\omega, \boldsymbol{k})} \left(\sum_\alpha e_\alpha \int \mathrm{d}^3 v \frac{G_\alpha(\boldsymbol{k}, \boldsymbol{v})}{\boldsymbol{k} \cdot \boldsymbol{v} - \omega} + \frac{k_{\|}(\Delta(\omega, \boldsymbol{k}) - 1)}{2\pi \mathscr{E}(\omega, k_{\|})} \times \int_{-\infty}^{\infty} \frac{\mathrm{d}k'_x}{k'^2 \Delta(\omega, \boldsymbol{k}')} \sum_\alpha e_\alpha \int \mathrm{d}^3 v \frac{G_\alpha(\boldsymbol{k}', \boldsymbol{v})}{\boldsymbol{k}' \cdot \boldsymbol{v} - \omega} \right) \tag{10}$$

where

$$\Delta(\omega, \boldsymbol{k}) = 1 + \frac{4\pi}{k^2} \sum_\alpha \frac{e_\alpha^2}{m_\alpha} \int \mathrm{d}^3 v \frac{\boldsymbol{k} \cdot (\partial f_{0\alpha}/\partial \boldsymbol{v})}{\omega - \boldsymbol{k} \cdot \boldsymbol{v}} \tag{11}$$

$$\mathscr{E}(\omega, k_{\|}) = 1 - \frac{k_{\|}}{2\pi} \int_{-\infty}^{\infty} \frac{dk_x}{k^2} \left(1 - \frac{1}{\Delta(\omega, \boldsymbol{k})} \right) \tag{12}$$

that if we neglect the influence of the initial perturbation considering the plasma state at a moment which is far enough away from the initial one, contributions to the inverse Laplace transform of the electric charge plasma perturbation (10) give only the residues at the roots of

$$\Delta(\omega, \boldsymbol{k}) = 0 \tag{13}$$

and

$$\mathscr{E}(\omega, k_{\|}) = 0. \tag{14}$$

Equations (13) and (14) are the dispersion relations of the propagating bulk and surface waves in the plasma respectively (Geurnsey 1969).

Taking into account the kind of equilibrium distribution function (3) and the equality

$$\frac{1}{\omega + i\gamma - \boldsymbol{k} \cdot \boldsymbol{v}} = -i \int_0^{\infty} \exp[i(\omega + i\gamma - \boldsymbol{k} \cdot \boldsymbol{v})\tau] \, d\tau \tag{15}$$

where γ is a small positive quantity, we obtain the following expression for $\Delta(\omega, \boldsymbol{k})$ (Ginzburg and Rukhadze 1970):

$$\Delta(\omega, \boldsymbol{k}) = 1 + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{k^2 v_{T\alpha}^2} \left[1 - J_+ \left(\frac{\omega - \boldsymbol{k} \cdot \boldsymbol{V}_\alpha}{|\boldsymbol{k}| v_{T\alpha}} \right) \right] \tag{16}$$

where $\omega_{p\alpha} = (4\pi e^2 n_0/m_\alpha)^{1/2}$ are the plasma frequencies of the particles, $v_{T\alpha} = (T_\alpha/m_\alpha)^{1/2}$ are their thermal velocities and

$$J_+ \left(\frac{\omega - \boldsymbol{k} \cdot \boldsymbol{V}_\alpha}{|\boldsymbol{k}| v_{T\alpha}} \right) = J_+(x_\alpha) = x_\alpha \exp\left(-\frac{x_\alpha^2}{2} \right) \int_{-i\infty}^{x_\alpha} dz \exp\left(\frac{z^2}{2} \right) = -i \left(\frac{\pi}{2} \right)^{1/2} x_\alpha W \left(\frac{x_\alpha}{\sqrt{2}} \right). \tag{17}$$

The values of the function $W(x_\alpha/\sqrt{2})$ one can take from the tables of Faddeeva and Terent'ev (1954).

We divide $\Delta(\omega, \boldsymbol{k})$ and $\mathscr{E}(\omega, k_{\|})$ into their real and imaginary parts:

$$\Delta(\omega, \boldsymbol{k}) = \Delta_1(\omega, \boldsymbol{k}) + i\Delta_2(\omega, \boldsymbol{k}) \tag{18}$$

$$\mathscr{E}(\omega, k_{\|}) = \mathscr{E}_1(\omega, k_{\|}) + i\mathscr{E}_2(\omega, k_{\|}) = 1 - \frac{k_{\|}}{\pi} \int_0^{\infty} \frac{dk_x}{k^2} \left(1 - \frac{\Delta_1 - i\Delta_2}{|\Delta(\omega, \boldsymbol{k})|^2} \right). \tag{19}$$

Further, we assume that $\omega = \omega_0 + i\gamma_\omega$ ($|\gamma_\omega/\omega_0| \ll 1$), where ω_0 is the frequency and γ_ω is the time damping rate of the surface waves, which are determined respectively by:

$$\mathscr{E}_1(\omega_0, k_{\|}) = 1 - \frac{k_{\|}}{\pi} \int_0^{\infty} \frac{dk_x}{k^2} \left(1 - \frac{\Delta_1(\omega, \boldsymbol{k})}{|\Delta(\omega, \boldsymbol{k})|^2} \right) = 0 \tag{20}$$

and

$$\gamma_\omega = -\frac{\mathscr{E}_2(\omega, k_{\|})}{(\partial \mathscr{E}_1/\partial \omega)_{\omega=\omega_0}} = \frac{1}{(\partial \mathscr{E}_1/\partial \omega)_{\omega=\omega_0}} \frac{k_{\|}}{\pi} \int_0^{\infty} \frac{dk_x}{k^2} \frac{\Delta_2(\omega, \boldsymbol{k})}{|\Delta(\omega, \boldsymbol{k})|^2}. \tag{21}$$

We shall investigate the spectrum of the high-frequency surface waves ($\omega^2 \gg \omega_{pi}^2$) in the long-wavelength approximation:

$$\frac{\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V}}{k_{\parallel} v_{Te}} \gg 1, \qquad \frac{\omega}{k_{\parallel} v_{Ti}} \gg 1 \qquad (\boldsymbol{k}_{\parallel} \cdot \boldsymbol{V} < \omega). \tag{22}$$

For the determination of $\mathscr{E}_1(\omega, k_{\parallel})$ and $\mathscr{E}_2(\omega, k_{\parallel})$ we use both asymptotes of the function $J_+(x_\alpha)$, the tabulated values of the probability function and the inequality $\Delta_1^2 \gg \Delta_2^2$, which is satisfied in the regions where one may use the asymptotes. The asymptote of the function $J_+(x_\alpha)$ with $x_{e,i} \gg 1$ (where $x_e = (\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})/|\boldsymbol{k}| v_{Te}$, $x_i = \omega/|\boldsymbol{k}| v_{Ti}$) is valid when $x_{e,i} \in (5, \infty)$ and with $x_{e,i} \ll 1$ when $x_{e,i} \in (0, \frac{1}{37})$. When $x_{e,i_1} = 5$ and $x_{e,i_2} = \frac{1}{37}$ the values of the function $J_+(x_{e,i})$, obtained from the asymptotes coincide with high enough precision with their tabulated values. The values of the wavenumber k_x (see equalities (20) and (21)) corresponding to $x_{e,i} = 1$ are

$$k_{x_2} = \{[(\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})^2/v_{Te}^2] - k_{\parallel}^2\}^{1/2} \qquad \text{and} \qquad k_{x_5} = [(\omega^2/v_{Ti}^2) - k_{\parallel}^2]^{1/2}$$

respectively, where $k_{x_2} < k_{x_5}$. Bearing in mind the kind of functions in the integrals in the expression for $\mathscr{E}_1(\omega, k_{\parallel})$ and $\mathscr{E}_2(\omega, k_{\parallel})$, we divide the integration region into the following intervals:

(i) $k_x \in (0, k_{x_1})$, where $k_{x_1} = \{[(\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})^2/x_{e_1}^2 v_{Te}^2] - k_{\parallel}^2\}^{1/2}$ and $x_{e,i} \gg 1$. Therefore in this case one can use the asymptote of $J_+(x_{e,i})$ for argument values $x_{e,i} \gg 1$.

(ii) $k_x \in (k_{x_1}, k_{x_3})$, where $k_{x_3} = \{[(\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})^2/x_{e_2}^2 v_{Te}^2] - k_{\parallel}^2\}^{1/2}$, $x_{e_2} = \frac{1}{37}$ in the expression for $\mathscr{E}_1(\omega, k_{\parallel})$ and $x_{e_2} = \frac{1}{11}$ in the expression for $\mathscr{E}_2(\omega, k_{\parallel})$. This interval is in the neighbourhood of the point k_{x_2}. As in this region $x_i \gg 1$, for the function $J_+(x_i)$ one can use its asymptote, while for the function $J_+(x_e)$ one uses the tabulated values.

(iii) $k_x \in (k_{x_3}, k_{x_4})$, where $k_{x_4} = [(\omega^2/x_{i_1}^2 v_{Ti}^2) - k_{\parallel}^2]^{1/2}$ and $x_e \ll 1$, $x_i \gg 1$. For the integration one uses asymptotes of the function $J_+(x_{e,i})$. Here $k_{x_3} < k_{x_4}$, since we consider a strongly non-isothermal plasma or a plasma with heavy ions.

(iv) $k_x \in (k_{x_4}, k_{x_6})$, where $k_{x_6} = [(\omega^2/x_{i_2}^2 v_{Ti}^2) - k_{\parallel}^2]^{1/2}$ ($x_{i_2} = \frac{1}{37}$ in $\mathscr{E}_1(\omega, k_{\parallel})$ and $x_{i_2} = \frac{1}{11}$ in $\mathscr{E}_2(\omega, k_{\parallel})$), is in the neighbourhood of the point k_{x_5}. In this region one can use the asymptotes of the function $J_+(x_e)$ for $x_e \ll 1$ and the tabulated values of the function $J_+(x_i)$.

(v) $k_x \in (k_{x_6}, \infty)$, where $x_{e,i} \ll 1$ and, therefore, one can use the asymptote of the function $J_+(x_{e,i})$ for small values of its arguments.

To derive the dispersion relation of the high-frequency surface waves, we transform the expression (20) to the form:

$$\mathscr{E}_1(\omega, k_{\parallel}) = \mathscr{E}_{10}(\omega, k_{\parallel}) + \mathscr{E}_{11}(\omega, k_{\parallel}) \tag{23}$$

where

$$\mathscr{E}_{10}(\omega, k_{\parallel}) = 1 + \frac{1}{2} \frac{\omega_{pe}^2}{(\omega - \boldsymbol{k} \cdot \boldsymbol{V})^2 - \omega_{pe}^2} \tag{24}$$

$$\begin{aligned}\mathscr{E}_{11}(\omega, k_{\parallel}) = {} & \frac{k_{\parallel}}{\pi}\left(1 - \frac{1}{1 - [\omega_{pe}^2/(\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})^2]}\right) \int_{k_{x_1}}^{\infty} \frac{dk_x}{k_x^2 + k_{\parallel}^2} \\ & + \frac{1}{\pi} \frac{k_{\parallel} v_{Te}}{\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V}} \frac{(\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})^2}{\omega_{pe}^2 - (\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})^2} \frac{\sqrt{3}}{\{1 - [(\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})^2/\omega_{pe}^2]\}^{1/2}} \\ & \times \tan^{-1}\left(\frac{3}{25\{1 - [(\omega - \boldsymbol{k}_{\parallel} \cdot \boldsymbol{V})^2/\omega_{pe}^2]\}}\right)^{1/2} - \frac{k_{\parallel}}{\pi} \int_{k_{x_1}}^{\infty} \frac{dk_x}{k_x^2 + k_{\parallel}^2}\left(1 - \frac{\Delta_1(\omega, \boldsymbol{k})}{|\Delta(\omega, \boldsymbol{k})|^2}\right)\end{aligned} \tag{25}$$

or

$$\mathscr{E}_{11}(\omega, k_\parallel) = \frac{1}{\pi}\frac{k_\parallel v_{\mathrm{Te}}}{\omega - \boldsymbol{k}_\parallel . \boldsymbol{V}}\frac{(\omega - \boldsymbol{k}_\parallel . \boldsymbol{V})^2}{\omega_{\mathrm{pe}}^2 - (\omega - \boldsymbol{k}_\parallel . \boldsymbol{V})^2}\frac{\sqrt{3}}{\{1 - [(\omega - \boldsymbol{k}_\parallel . \boldsymbol{V})^2/\omega_{\mathrm{pe}}^2]\}^{1/2}}$$
$$\times \tan^{-1}\left(\frac{3}{25\{1 - [(\omega - \boldsymbol{k}_\parallel . \boldsymbol{V})^2/\omega_{\mathrm{pe}}^2]\}}\right)^{1/2} + \frac{k_\parallel}{\pi}\int_{k_{x1}}^{\infty}\frac{\mathrm{d}k_x}{k_x^2 + k_\parallel^2}$$
$$\times \left(\frac{\Delta_1(\omega, \boldsymbol{k})}{|\Delta(\omega, \boldsymbol{k})|^2} - \frac{1}{1 - [\omega_{\mathrm{pe}}^2/(\omega - \boldsymbol{k}_\parallel . \boldsymbol{V})^2]}\right). \tag{26}$$

These expressions are valid when the inequalities: $k_\parallel \neq 0$, $(\omega - \boldsymbol{k}_\parallel . \boldsymbol{V})^2 \neq \omega_{\mathrm{pe}}^2$ and $(\omega - \boldsymbol{k}_\parallel . \boldsymbol{V})^2 \neq \omega_{\mathrm{pe}}^2 + 3k_\parallel^2 v_{\mathrm{Te}}^2$, are fulfilled.

The expression $\mathscr{E}_{11}(\omega, k_\parallel)$ tends to zero when $k_\parallel v_{\mathrm{Te}}/(\omega - \boldsymbol{k}_\parallel . \boldsymbol{V}) \to 0$ and $k_\parallel v_{\mathrm{Ti}}/\omega \to 0$ and, therefore, in the considered frequency region $\mathscr{E}_{11}(\omega, k_\parallel) \ll \mathscr{E}_{10}(\omega, k_\parallel)$. From this inequality it follows that the frequency of the surface waves has to be:

$$\omega_0 = \omega_{00} + \omega_{01}, \qquad (\omega_{01} \ll \omega_{00}) \tag{27}$$

where ω_{00} is determined from the equation

$$\mathscr{E}_{10}(\omega, k_\parallel) = 0 \tag{28}$$

and ω_{01}, which includes contributions of the thermal motion of the particles, is determined from the expression:

$$\omega_{01} = -\frac{\mathscr{E}_{11}(\omega_{00}, k_\parallel)}{(\partial\mathscr{E}_{10}/\partial\omega)_{\omega=\omega_{00}}}. \tag{29}$$

4. Results and discussion

The method described leads to the following dispersion relation and time damping rate of the propagating surface waves:

$$\omega_0 = \frac{\omega_{\mathrm{pe}}}{\sqrt{2}}\left[1 + C_1 k_\parallel r_{\mathrm{De}}\left(1 + C_2\frac{v_{\mathrm{Ti}}}{v_{\mathrm{Te}}}\frac{\omega_{\mathrm{pe}}}{(\omega_{\mathrm{pe}}/\sqrt{2}) + \boldsymbol{k}_\parallel . \boldsymbol{V}}\right)\right] + \boldsymbol{k}_\parallel . \boldsymbol{V} \tag{30}$$

$$\gamma_\omega = -\frac{\omega_{\mathrm{pe}}}{\sqrt{2}}k_\parallel r_{\mathrm{De}} C_3\left[1 - A\left(\frac{v_{\mathrm{Ti}}}{v_{\mathrm{Te}}}\right)\right], \tag{31}$$

where $r_{\mathrm{De}} = (T_{\mathrm{e}}/4\pi e^2 n_0)^{1/2}$ is the electron Debye length, $C_1 = 1{\cdot}22$, $C_2 = 1{\cdot}30$, $C_3 = 0{\cdot}177$ and $A(v_{\mathrm{Ti}}/v_{\mathrm{Te}})$ is a small correction depending on the ratio $v_{\mathrm{Ti}}/v_{\mathrm{Te}}$:

$$A\left(\frac{v_{\mathrm{Ti}}}{v_{\mathrm{Te}}}\right) = \frac{v_{\mathrm{Ti}}}{v_{\mathrm{Te}}}\frac{\omega_{\mathrm{pe}}}{(\omega_{\mathrm{pe}}/\sqrt{2}) + \boldsymbol{k}_\parallel . \boldsymbol{V}}\left(C_4 - C_5\frac{v_{\mathrm{Ti}}}{v_{\mathrm{Te}}}\frac{\omega_{\mathrm{pe}}}{(\omega_{\mathrm{pe}}/\sqrt{2}) + \boldsymbol{k}_\parallel . \boldsymbol{V}} - C_6\frac{\omega_{\mathrm{pi}}^2}{[(\omega_{\mathrm{pe}}/\sqrt{2}) + \boldsymbol{k}_\parallel . \boldsymbol{V}]^2}\right) \tag{32}$$

where $C_4 = 4{\cdot}66 \times 10^{-3}$, $C_5 = 9{\cdot}77$, $C_6 = 2{\cdot}25$.

The values obtained for the coefficients C_1 and C_3 calculated with enough precision coincide with those, indicated in the work by Aliev *et al* (1972), for a plasma without current. The value obtained for the coefficient C_1 differs essentially from that derived in the work by Guernsey (1969), because in the latter paper:

(*a*) the first term in the expression (25), which is a correction due to the thermal effects in the dispersion relation of the surface waves, is not taken into account. This term is obtained as a contribution from the region of integration (i) in the expression for $\mathscr{E}_1(\omega, k_{\parallel})$. It gives practically a principal contribution with the calculation of C_1.

(*b*) Only the asymptotes of the probability function are used. They are applied also in regions (ii) and (iv) where they are not valid. The supposition that the contributions to the value of C_1 from the neighbourhoods of the points

$$k_{x_2} = \{[(\omega - \boldsymbol{k}_{\parallel} \,.\, \boldsymbol{V})^2/v_{Te}^2] - k_{\parallel}^2\}^{1/2} \qquad \text{and} \qquad k_{x_5} = [(\omega^2/v_{Ti}^2) - k_{\parallel}^2]^{1/2}$$

are negligibly small is used without any justification.

The difference between the values of the coefficient C_3 obtained by Aliev *et al* (1972) (on the basis of the work by Romanov 1964) and us and by Guernsey (1969) is due to the fact that in the latter work only the asymptotes of the probability function have been used.

With $k_{\parallel} \neq 0$ the dispersion equation for surface waves, derived by Guernsey (1969), coincides with the electrostatic approximation of the dispersion equation obtained by Romanov (1964); one can see that from the expressions (12) and (23)–(26).

Because of the condition $\omega \gg \omega_{pi}$ a growth of the high-frequency surface waves because of the presence of a current in the plasma is not possible (it is not possible that $\gamma_\omega > 0$).

One can see from the expression (30) that a Doppler shift of the frequency of the propagating surface waves appears because of the presence of the directed electron velocity in the plasma.

When the waves propagate along the current ($\cos(\boldsymbol{k}_{\parallel}, \boldsymbol{V}) = 1$), the phase velocity $v_{ph} = C_7 v_{Te} + |\boldsymbol{V}| + (\omega_{pe}/\sqrt{2}k_{\parallel})$, where $C_7 = C_1/\sqrt{2} = 0{\cdot}863$ and the group velocity $V_{gr} = C_7 v_{Te} + |\boldsymbol{V}|$ of the waves has the same direction as the current (forward downstream waves). When $\cos(\boldsymbol{k}_{\parallel}, \boldsymbol{V}) = -1$ the phase velocity $v_{ph} = C_7 v_{Te} - |\boldsymbol{V}| + (\omega_{pe}/\sqrt{2}k_{\parallel})$ of the wave has a direction opposite to the drift velocity of the electrons (upstream waves). In this case, if $C_7 v_{Te} > |\boldsymbol{V}|$ the group velocity is in the same direction as the phase velocity (forward upstream waves) and if $C_7 v_{Te} < |\boldsymbol{V}|$ the group velocity is directed along the current, ie the waves have opposite phase and group velocities (backward upstream waves).

Therefore the expression (30) describes: (i) a spectrum of forward waves (downstream and upstream) which can exist in the frequency region $\omega_0 > \omega_{pe}/\sqrt{2}$. For the propagation of upstream waves, the velocities v_{Te} and $|\boldsymbol{V}|$ must satisfy the condition $C_7 v_{Te} > |\boldsymbol{V}|$. (ii) a spectrum of backward upstream waves, propagating in the frequency region $\omega_0 < \omega_{pe}/\sqrt{2}$ when $C_7 v_{Te} < |\boldsymbol{V}|$.

If $C_7 v_{Te} = |\boldsymbol{V}|$ the upstream waves transform into localized oscillations at a frequency $\omega_0 = \omega_{pe}/\sqrt{2}$ and, therefore, in this case only downstream waves propagate in the plasma.

From the comparison of the wavenumbers of the forward waves one establishes that the downstream wavenumber is smaller than the upstream wavenumber. This confirms the experimental results, reported by Aničin *et al* (1973).

For the space damping rate $\gamma_k = \operatorname{Im} k = -\gamma_\omega/V_{gr}$ we obtain the following expression:

$$\gamma_k = \frac{\omega_{pe}}{\sqrt{2}} k_{\parallel} r_{De} \frac{C_3}{C_7 v_{Te} \pm |\boldsymbol{V}|} \left[1 - A\left(\frac{v_{Ti}}{v_{Te}}\right)\right]. \tag{33}$$

The space damping rate is different for the downstream and upstream waves. For the forward waves $\gamma_k > 0$ and this value corresponds to the wave damping (the perturbations have the form $\exp(-\mathrm{i}\omega t + \mathrm{i}\boldsymbol{k}_{\parallel} \,.\, \boldsymbol{r})$). Comparing the space damping rate of the waves in this case we conclude that γ_k for the downstream waves is less than the one for the upstream waves, which is in an agreement with the experimental results reported by Aničin *et al* (1973). For the backward waves we obtain $\gamma_k < 0$ which also corresponds to a wave damping (in this case the wavevector direction $\boldsymbol{k}_{\parallel}$ and the direction of propagation of the wave energy $\boldsymbol{r}$ are opposite and, therefore, the perturbations are $\sim \exp(\gamma_k|\boldsymbol{r}|)$ with $\gamma_k < 0$).

For the ratio δ/λ, where $\lambda = 2\pi/k_{\parallel}$ is the wavelength and δ is the distance over which the wave amplitude falls by a factor e, we obtain:

$$\frac{\delta}{\lambda} = \begin{cases} \dfrac{\sqrt{2}}{2\pi C_3}\left(C_7 + \dfrac{|V|}{v_{\mathrm{Te}}}\right) & \text{for forward downstream waves} \\ \dfrac{\sqrt{2}}{2\pi C_3}\left(C_7 - \dfrac{|V|}{v_{\mathrm{Te}}}\right) & \text{for forward upstream waves} \\ \dfrac{\sqrt{2}}{2\pi C_3}\left(\dfrac{|V|}{v_{\mathrm{Te}}} - C_7\right) & \text{for backward upstream waves.} \end{cases}$$

One may see that the damping constant δ/λ for the upstream waves is less than the one for downstream waves.

In conclusion we mention that experiments have recently begun at Sofia University to test the theory presented here. The experimental results and the theory for a more realistic plasma geometry will be presented in a subsequent paper.

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